KSME Journal, Vol. 9, No. 1, pp. 102~114, 1995

Stable Joint Torque Optimization for Multiple Cooperating Redundant Manipulator System

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(Received August 8, 1994)

In this paper, joint torque optimization for multiple cooperating redundant manipulators rigidly handling a common object is considered. This work focuses on finding the optimal and stable distribution of the operational forces of a multiple redundant manipulator system to the individual manipulators. Two joint torque optimization schemes(local joint torque minimization and natural joint motion) are formulated and compared. When a redundant manipulator with its joints free is driven by its tip, a naturally inducing joint motion can be called 'natural joint motion'. From the simulation results of a system of three cooperating redundant manipulators, the natural joint motion scheme is shown to be better than the local joint torque minimization scheme with regard to global torque minimization capability and the resulting stability of motion. However, in order to guarantee the stability, the null space damping method is required for the both schemes. The effectiveness of the null space damping method is demonstrated by simulation. Additionally, the condition for the distribution of the operational forces required to drive the given system along a natural joint motion trajectory is addressed.

Key Words : Joint Torque Optimization, Multiple Cooperating Manipulators, Null Space Damping Method, Natural Joint Motion

1. Introduction

Cooperative use of multiple manipulators(or fingers) will allow the performance of more industrial applications than can currently be undertaken using single manipulators. Such systems can provide greater load capacity, better manipulation capability, and higher flexibility in automated manufacturing. Typical example applications include transport of heavy material, fine manipulation of objects and part assembly. A number of works dealing with cooperative execution of tasks performed by multiple cooperating manipulators have appeared recently. One of the main topics of this research is the problem of load distribution.

The load distribution techniques for multiple cooperating manipulator systems handling a common object appearing in the literature may be categorized into three groups : 1) Dynamic loads of the common object are determined and distributed to the tip of each individual manipulator according to a number of different criteria involving contact geometry between the common object and the manipulators. Then, the optimally distributed forces/moments are directly added to the dynamics of each manipulator by using the transpose of Jacobian. This technique emphasizes the object to be manipulated rather than the manipulators and has been mainly used for fine manipulation of the object(Kerr and Roth, 1986 ; Kumar and Waldron, 1987). 2) Dynamic loads of the common object are determined and directly incorporated into the dynamics of each manipulator according to task dependent performance criteria involving the total system geometry. This technique has been mainly used for transport of a heavy object(Zheng and Luh, 1988 ; Pittelkau, 1988). 3) Reduced order dynamic models of the total system are expressed in terms of the task

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coordinate set, or any system independent coordinate set, by embedding the closed chain constraints into the motion of the system. The model based dynamic loads required to complete the given task are distributed according to cost saving criteria involving the total system geometry. This technique has also been used for transport of a heavy object(Carignan and Akin, 1989; Hsu, 1989; Khatib, 1988; Kreutz and Lokshin, 1988).

Most of works, except Hu and Goldenburg(1990), are not concerned with the utilization of the redundancy of each manipulator with respect to the given task motion. In this paper, local joint torque optimizations for multiple cooperating redundant manipulator system are performed, which might belong to the third load distribution category above. The method starts with the operational space dynamic formulation for multiple cooperating redundant manipulators. From this formulation, the unique matrix is obtained, which maps the joint torque set of the individual manipulators onto its corresponding task-dependent operational forces. Based on this functional relationship, two joint torque optimization schemes(local joint torque minimization and natural joint motion) are formulated. The stability problem, commonly encountered in local joint torque optimization of single redundant manipulators, was also observed in the multiple cooperating redundant manipulator system through the simulation of three 3 DOF cooperating manipulators for planar translational operational trajectory. In addition, the stability of algorithms using the transpose of Jacobian among load distribution schemes above can not be guaranteed in cased of redundant situation, in order to map the optimally distributed operational forces(or contact forces/moments) to each individual manipulator. The instability is shown due that the use of the transpose of Jacobian for distribution of the operational forces leads to the natural joint motion, which often tends to be unstable for a long time trajectory. In order to eliminate the stability problem, the null space damping method(Kang and Freeman, 1993) is considered for the multiple cooperating redundant manipulator system. The effectiveness of the null space damping method is demonstrated by simulation.

2. Operational Space Dynamic Modeling of Multiple Cooperating Redundant Manipulators Rigidly Handling a Common Object

The configuration of the considered system is shown in Fig. 1. The kinematic and dynamic model of each individual manipulator is assumed known. First, the object dynamics is considered as follows, the Newton-Euler equations for the object are

$$f = m_o \ddot{r} - m_o g \quad (1)$$

$$n = [I_o]\dot{w} + w \times ([I_o] w) \quad (2)$$

where m_o and $[I_o]$ are the mass and inertia matrix of the object and r and w are the position of the operational point and the absolute angular velocity of the object, respectively.

Equations (1) and (2) are combined to yield the operational forces required to move the object as

$$F_o = [{}_oI^*_{uu}]\ddot{u} + {}_oC_u \quad (3)$$

where

$$[{}_oI^*_{uu}] = \begin{bmatrix} m_o[I] & [0] \\ [0] & I_o \end{bmatrix} \quad (4)$$

with the 3×3 identity matrix and null matrix being $[I]$ and $[0]$, respectively

and

$$\ddot{u} = \begin{bmatrix} \ddot{r} \\ \dot{w} \end{bmatrix}; \; {}_oC_u = \begin{bmatrix} -m_o g \\ w \times (I_o w) \end{bmatrix}; \; F_o = [f^T, n^T]$$

Now, the operational space dynamic model of each individual manipulator($i = 1, 2, \cdots, L$) is

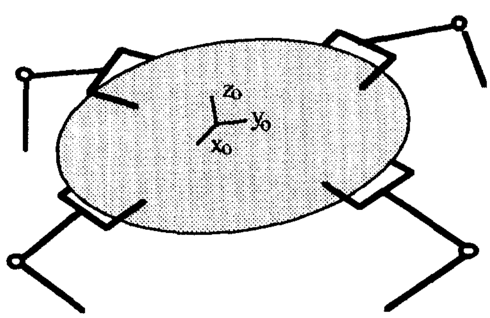

Fig. 1 Multiple cooperating redundant manipulators handling a common object

considered based on its joint space dynamic model and the Jacobian, J_i, which relates the Cartesian velocity of the center of mass of the object to the joint coordinate velocities of the i^{th} manipulator. The joint space dynamic model of the i^{th} manipulator is expressed as

$$ {}_i\tau_\phi=[{}_iI^*_{\phi\phi}]\ddot{\phi}_i+C(\phi_i,\ \dot{\phi}_i) \quad i=1,\ 2,\ \cdots,\ L \tag{5}$$

where ϕ_i is joint coordiate vector of the i^{th} manipulator with ${}_i\tau_\phi$, $[{}_iI^*_{\phi\phi}]$ and $C(\phi_i,\ \dot{\phi}_i)$ being its $n\times 1$ joint load vector, $n\times n$ inertia matrix and $n\times 1$ vector of Coriolis, centripetal and gravity components, respectively.

The joint space dynamic model can be transferred to the operational space dynamic model according to the formulation given in Appendix 1. The effective operational forces due to the i^{th} manipulator can be written as

$$F_i=[{}_iI^*_{uu}]\ddot{u}+{}_iC_u \tag{6}$$

where the $m\times m$ effective inertia matrix and the $m\times 1$ vector of Coriolis, centripetal and gravity components of the i^{th} manipulator are, respectively,

$$[{}_iI^*_{uu}]=(J_i[{}_iI^*_{\phi\phi}]^{-1}J_i^T)^{-1} \tag{7}$$

and

$$\begin{aligned} {}_iC_u= &-[{}_iI^*_{uu}]H(\phi_i,\ \dot{\phi}_i) \\ &+({}_iJ_i^+)^T C(\phi_i,\ \dot{\phi}_i) \\ \text{with } &({}_iJ_i^+)^T=[{}_iI^*_{uu}]J_i[{}_iI^*_{\phi\phi}]^{-1} \end{aligned} \tag{8}$$

where $H(\phi_i,\ \dot{\phi}_i)$ is Coriolis and centripetal acceleration vector of the i^{th} manipulator.

The operational forces of the considered robotic system can be expressed as a sum of effective operational force contribution of each individual manipulator and the common object as

$$F=F_o+\sum_{i1}^{L}F_i=[I^*_{uu}]\ddot{u}+C_u \tag{9}$$

where the corresponding mxm inertia matrix of the given robotic system is

$$[I^*_{uu}]=[{}_oI^*_{uu}]+\sum_{i=1}^{L}[{}_iI^*_{uu}] \tag{10}$$

with the $m\times 1$ vector of Coriolis, centripetal and gravity components being

$$C_u={}_oC_u+\sum_{i=1}^{L}{}_iC_u \tag{11}$$

3. Functional Relationship Between Joint Torques and Operational Forces

Using Eqs. (9)~(11), the operational forces required to drive the given robotic system along the desired operational space trajectory are obtained from the control structure of the given robotic system. The required joint torque is not unique. The surjective function mapping the joint torque set onto its effective operational forces can be represented by augmenting the matrices, $({}_iJ_i^+)^T$, seen in Eq. (8).

Proposition 1) Any set of joint torques of multiple redundant manipulators rigidly handling a common object can be mapped onto its effective operational forces by a unique transformation matrix, $[J_I^+]^T$.

$$F=[J_I^+]^T\tau_\phi$$

with

$$[J_I^+]^T=\left[({}_1J_I^+)^T:\cdots:({}_iJ_I^+)^T:\cdots({}_LJ_I^+)^T\right] \tag{12}$$

where the joint torque set

$$\tau_\phi=[{}_1\tau_\phi^T\cdots{}_i\tau_\phi^T\cdots{}_L\tau_\phi^T]^T$$

and

$$({}_iJ_I^+)^T=[{}_iI^*_{uu}]J_i[{}_iI^*_{\phi\phi}]^{-1}$$

Proof: See Appendix 1.

The next two propositions are given to illustrate the condition for the distribution of operational forces to a joint torque set of the multiple cooperating redundant manipulator system for inducing natural joint motion. This condition might help one to understand the natural behavior of the total system and will be used for a joint torque optimization scheme to follow.

Proposition 2) For a given operational force vector(F), a system independent joint torque set(τ_a) which is mapped by a transformation matrix, $(J_a^u)^T$, drives the multiple cooperating redundant manipulator system along the natural joint motion, where the Jacobian matrix $(J_a^u)^T$ relates the system's operational space velocity vector to the system independent joint velocity vector. (For clarity, see Eq. (19)).

Proof: When the multiple manipulators hold a common object, closed chain mechanisms are formed. Due to the closed chain constraint equations, the Lagrangian coordinates employed to initially describe the system are divided into a system independent coordinate set and a system dependent coordinate set. Then, the equations of motion of the given system can be completely described in terms of system independent coordinate set. Once the minimal kinematic and dynamic models are obtained, the argument for the natural joint motion of the multiple cooperating manipulator system can be treated in the same fashion as that for a single redundant manipulator. From the torque solution in Eq. (12) of Kang and Freeman(1993), the proposition is evident.

(Q. E. D)

Proposition 3)

$$_i\tau_\phi = J_i^T F_i \quad i=1, 2, \cdots, L \tag{13}$$

and

$$F = F_1 + F_2 + \cdots + F_L. \tag{14}$$

Any joint torque set obtained from the operational force distribution according to Eqs. (13) and (14) drives the multiple cooperating redundant manipulator system along the natural joint motion.

Proof: The system independent actuating torque set, τ_a, is effectively equivalent to the corresponding Lagrangian torque set, τ_ϕ, in that both joint torque sets drive the system along the same joint trajectories. The relationship is written as

$$\tau_a = (J_a^\phi)^T \tau_\phi \tag{15}$$

where (J_a^ϕ) is Jacobian matrix relating the system Lagrange coordinate set and the system independent coordinate set.

The detailed expression of Eq. (15) can be written as

$$\tau_a = [({}_1J_a^\phi)^T \cdots ({}_iJ_a^\phi)^T \cdots ({}_LJ_a^\phi)^T] \begin{bmatrix} {}_1\tau_\phi \\ \vdots \\ {}_i\tau_\phi \\ \vdots \\ {}_L\tau_\phi \end{bmatrix} \tag{16}$$

where $({}_iJ_a^\phi)$ is the Jacobian matrix relating the system Lagrange coordinate set of the i^{th} manipulator and the system independent set(Refer to Kang and Freeman(1994) for generation of the Jacobians). Substituting Eq. (13) into Eq. (16) yields

$$\begin{aligned} \tau_a &= [({}_1J_a^\phi)^T \cdots ({}_iJ_a^\phi)^T \cdots ({}_LJ_a^\phi)^T] \begin{bmatrix} J_1^T F_1 \\ \cdots \\ J_i^T F_i \\ \cdots \\ J_L^T F_L \end{bmatrix} \\ &= ({}_1J_a^\phi)^T J_1^T F_1 \\ &\quad + \cdots + ({}_iJ_a^\phi)^T J_i^T F_i + \cdots \\ &\quad + ({}_LJ_a^\phi)^T J_L^T F_L. \end{aligned} \tag{17}$$

The Jacobian matrix relating the operational space velocity vector to the system independent coordinate velocity vector is obtained by substituting the internal joint kinematic relationship($i.\ e.$, J_a^ϕ) into the first order kinematics of a selected manipulator. Regardless of which manipulator is taken to describe the operational space motion, the same Jacobian matrix is obtained. Therefore,

$$(J_a^u) = J_1({}_1J_a^\phi) = \cdots = J_i({}_iJ_a^\phi) = \cdots = J_L({}_LJ_a^\phi). \tag{18}$$

Combining Eqs. (13), (16) and (17) yields

$$\tau_a = (J_a^u)^T F. \tag{19}$$

From Proposition 2), the joint torque set obtained from this distribution scheme drives the system along the natural joint motion. (Q.E.D)

4. Joint Torque Optimization

Equation (12) is a relationship between the manipulator joint torques and the states of the system which are specified by the trajectory of the common object. What remains is to specify the manipulator joint torques required to achieve the specified operational trajectory of the object grasped by a group of redundant manipulators. The underdetermined problem of solving the joint torques from Eq. (12) allows one to optimize a given performance criterion as an additional control constraint, which then uniquely determines the joint torques. In this section, two local joint torque optimization schemes are treated, based on the functional relationship given in Eq. (12).

They are the joint torque minimization scheme and the natural joint motion scheme. The resulting torques of both schemes given here include fictitious damping forces acting in the null space of $[J_I^+]^T$, since stability problems are expected here as they were in the joint torque optimization schemes(Kang and Freeman, 1993) for single redundant manipulators. In that paper, the condition for null space damping matrix to achieve the positive damping is addressed.

4.1 Joint torque minimization(JTM)

$$\mathrm{Min}(\frac{1}{2}\tau_\phi^T\tau_\phi)$$
subject to $F=[J_I^+]^T\tau_\phi$.

Incorporating null space dissipative forces, the command torques become

$$\tau_\phi = P_2F - K_d(I-P_2[J_I^+]^T)\dot{\phi} \tag{20}$$

where

$$P_2=[J_I^+]([J_I^+]^T[J_I^+])^{-1}. \tag{21}$$

The particular solution seen in Eq. (20) provides the local minimum joint torque norm required to drive the total system along a specified operational trajectory.

4.2 Natural joint motion(NJM)

As seen in Nedungadi and Kazerounian(1989), the joint torque optimization scheme may be obtained using the inverse inertia weighted joint torque norm as an objective function :

$$\mathrm{Min}\left(\frac{1}{2}\tau_\phi^T[I_{\phi\phi}^*]^{-1}\tau_\phi\right)$$
subject to $F=[J_I^+]^T\tau_\phi$

where the inverse inertia matrix is constructed such that its i^{th} diagonal submatrix is the inverse of the joint inertia matrix of the i^{th} manipulator. The command torques, including the null space component, in terms of the weighted Moore-Penrose generalized inverse, is obtained as

$$\tau_\phi = P_1F \tag{22}$$

where

$$P_1=[I_{\phi\phi}^*][J_I^+]([J_I^+]^T[I_{\phi\phi}^*][J_I^+])^{-1} \tag{23}$$

and the inertia matrix, $[I_{\phi\phi}^*]$, is constructed such that its ith diagonal submatrix is the joint inertia matrix of the i^{th} manipulator.

Decoupling Eq.(22), the joint torques of the i^{th} manipulator can be expressed as

$$_i\tau_\phi = J_i^T[_iI_{uu}^*][I_{uu}^*]^{-1}F. \quad i=1,2,\cdots,L \tag{24}$$

It is recognized from proposition 3) that the joint torque solution in Eq. (24) drives the given system along natural joint motion(here, $F_i=[_iI_{uu}^*][I_{uu}^*]^{-1}F$ in Eq. (13)). Equation (24) also shows that the operational forces are distributed to the individual manipulators according to their effective operational inertia contribution. It is also shown that the effective operational inertia matrix, obtained from the object dynamics, has nothing to do with this distribution scheme. Although the resulting torques drive the system along natural joint motion, the fact that the joint torque set corresponding to a given joint motion is not unique due to the additional force redundancy of multiple cooperating redundant manipulators, give rises to a question : Does the operational inertia distribution scheme generates the true minimum norm among all other possible sets of joint torques which can drive the given system along the natural joint motion? This question leads to the consideration of the following joint torque optimization scheme. It is assumed that the given system makes natural joint motion. With the aid of Proposition 3), the joint torque optimization scheme for natural joint motion is reformulated as

$$\mathrm{Min}\left(\frac{1}{2}\tau_\phi^T\tau_\phi\right)$$
subject to $_i\tau_\phi=J_i^TF_i \; i=1,2,\cdots,L$

$$F=\sum_{i=1}^{L}F_i. \tag{25}$$

Using the $L\times1$ vector of Lagrange multipliers, λ, and substituting Eq. (25) into the objective function, the augmented objective function, I, can be written as

$$I=\sum_{i1}^{L}\frac{1}{2}F_i^TJ_iJ_i^TF_i+\lambda^T(F-\sum_{i=1}^{L}F_i). \tag{26}$$

The problem is now reduced to determining the minimum solution of the unconstrained objective function, I, in terms of the independent parameters, λ and F_i ($i=1, 2, \cdots, L$). This requires that

$$\frac{\partial I}{\partial F_i}=J_iJ_i^TF_i-\lambda=0 \quad i=1, 2, \cdots, L \tag{27}$$

and

$$\frac{\partial I}{\partial \lambda}=F-\sum_{i=1}^{L}F_i=0. \tag{28}$$

Assuming that there is no kinematic degeneracy in any of the individual manipulators, solving Eq. (27) for F_i yields

$$F_i=(J_iJ_i^T)^{-1}\lambda. \quad i=1, 2, \cdots, L \tag{29}$$

Substituting Eq. (29) into (28) and solving for λ gives

$$\lambda=\{\sum_{i=1}^{L}(J_iJ_i^T)^{-1}\}^{-1}F. \tag{30}$$

Substituting Eq. (30) into (29) yields

$$F_i=(J_iJ_i^T)^{-1}\{\sum_{i1}^{L}(J_iJ_i^T)^{-1}\}^{-1}F$$
$$i=1, 2, \cdots, L. \tag{31}$$

Substituting Eq. (31) into (25), the joint torques required to satisfy the objective function are

$${}_i\tau_\phi=J_i^T(J_iJ_i^T)^{-1}\{\sum_{i=1}^{L}(J_iJ_i^T)^{-1}\}^{-1}F.$$
$$i=1, 2, \cdots, L \tag{32}$$

From Appendix 2., the physical interpretation of the force distribution of Eq. (31) is that the operational forces are distributed to the individual manipulators according to their effective operational stiffness contribution, for the case in which all joints of the given system are assumed to have equal motor stiffness. Based on this physical interpretation, Eq. (32) can be written as

$${}_i\tau_\phi=J_i^T[{}_iK_{uu}^*][{}_iK_{uu}^*]^{-1}F \quad i=1, 2, \cdots, L \tag{33}$$

where the effective operational linear stiffness matrix of the i^{th} manipulator is

$$[{}_iK_{uu}^*]=(J_iJ_i^T)^{-1} \quad i=1, 2, \cdots, L \tag{34}$$

with the total effective operational linear stiffness matrix of the system being by

$$[K_{uu}^*]=\sum_{i=1}^{L}(J_iJ_i^T)^{-1}. \tag{35}$$

This solution shows that the local torque minimum for natural joint motion is obtained from the distribution of the operational forces to the individual manipulators according to their operational stiffness contribution rather than their operational inertia contribution. Incorporating the null space damping forces, which satisfy the stability condition(seen in Kang and Freeman, 1993), the command torques from this scheme become

$${}_i\tau_\phi=J_i^T[{}_iK_{uu}^*][{}_iK_{uu}^*]^{-1}F$$
$$-K_d\{[I]-J_i^T(J_i^+)^T\}[{}_iI_{\phi\phi}^*]\dot{\phi}_i.$$
$$i=1, 2, \cdots, L \tag{36}$$

In this section, the joint torque minimization and natural joint motion schemes are presented for the desired performance : the resulting torques are globally stable and have good torque minimization capability. The performances of the joint torque minimization scheme(Eq. (20)) and the natural joint motion scheme(Eq. (36)) will be compared through the following simulation results.

5. Numerical Simulation and Discussion

The joint torque minimization and natural joint motion schemes, for both the undamped and damped cases, are simulated for a long time circular trajectory. In this example, the simulated manipulator system, seen in Fig 2, consists of

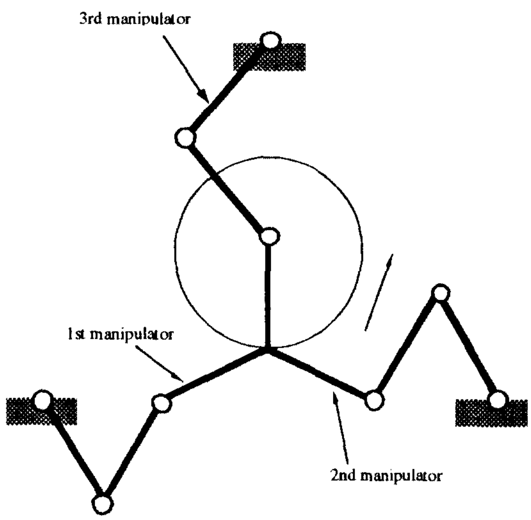

Fig. 2 Three cooperating redundant manipulators to the X-Y task space

three cooperating 3R planar manipulators operating in X-Y task space without gravity. Each link is modeled as a thin uniform rod with a length of 1.0 m and a mass of 10 kg. The simulated desired hand trajectory is a circular path with high acceleration as followings : radius $=$ 0.5 m, $\ddot{x} = -\text{radius}\ \pi^2\cos(\pi t)$ m/s^2, and $\ddot{y} = -\text{radius}\ \pi^2\sin(\pi t)$ m/s^2. The total simulation interval is 4 sec, completing 2 periodic cycles. The initial state of the manipulator is $\phi=[\phi_1^T, \phi_2^T, \phi_3^T]^T=[-64.34, 128.68, -34.34, 115.66, 128.68, -94.34, -131.41, 82.82, -41.41]^T$(deg) and $\dot{\phi}=[\dot{\phi}_1^T, \dot{\phi}_2^T, \dot{\phi}_3^T]^T=[0.812, -1.789, 1.060, -0.977, 1.788, -0.729, 0.992, 0, 0.909]^T$(rad/sec). The manipulator dynamics are integrated at an interval of 0.01 sec with the fourth order Runge Kutta integration routine. For a forward dynamic simulation of the three cooperating manipulators, the dynamic models in terms of system generalized coordinate set, are required. First, the optimal joint torque sets, τ_ϕ, are obtained from either Eq. (20) or Eq. (36) according to the employed joint torque optimization scheme. Recalling Eq. (15), the effectively equivalent joint torques, τ_a, are obtained according to the selected minimum set of coordinates(ϕ_a)(selected as the three base joint coordinates of each manipulator in this example). Then, the joint accelerations are evaluated by solving the dynamic equation based on the selected minimum set of coordinates. The simulation results of the joint velocity norm, the acceleration norm, and the torque norm trajectories are given for each algorithm, and for both the undamped and damped cases. The units of the plotted velocity, acceleration and torque norms are (rad/sec)^2, (rad/sec^2)^2 and (Nm)^2, respectively.

The simulation results, seen in Figs. (3)~(14), show generally similar characteristics to the results of the single redundant manipulator case in Kang and Freeman(1993) with respect to the global torque minimization capability, the resulting stability, and the convergence to a certain null space damped joint trajectory. From comparison of Figs. (3)~(5) and Figs. (9)~(11), however, it is noted that the undamped NJM scheme generally leads to stable motion and torque trajectories during the simulation interval(4 sec), while the

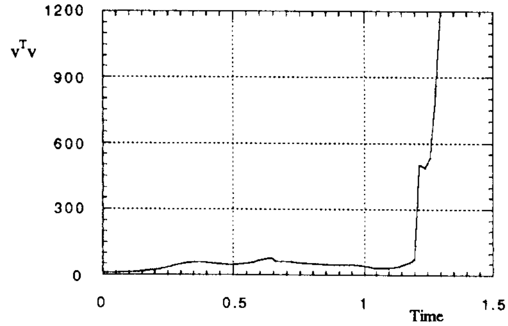

Fig. 3 Velocity norm trajectory of undamped JTM(K_d=0)

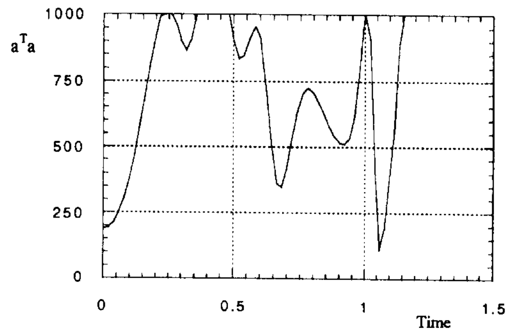

Fig. 4 Acceleration norm trajectory of undamped JTM(K_d=0)

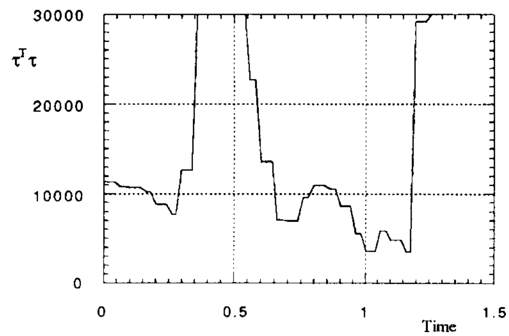

Fig. 5 Torque norm trajectory of undamped JTM (K_d=0)

undamped JTM scheme easily deviates from the desired system performance. For the single redundant manipulator case, both undamped schemes show the same level of stability problems. The global characteristics(i. e., the global minimization of the constrained Lagrangian) of the undamped NJM scheme are prominently illustrated

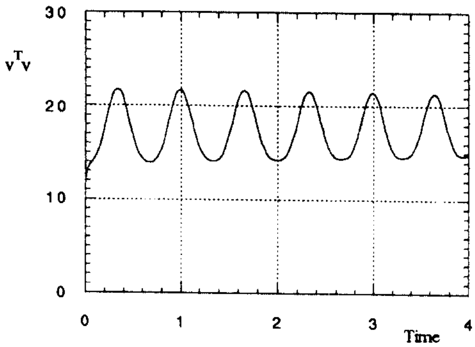

Fig. 6 Velocity norm trajectory of null space damped JTM(K_d=200)

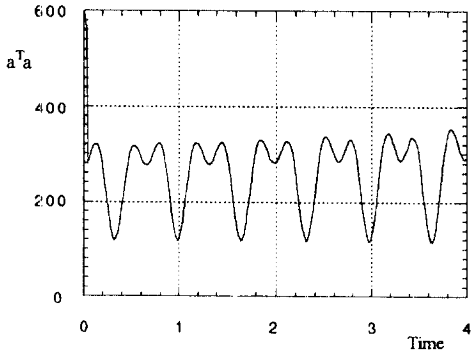

Fig. 7 Acceleration norm trajectory of null sace damped JTM(K_d=200)

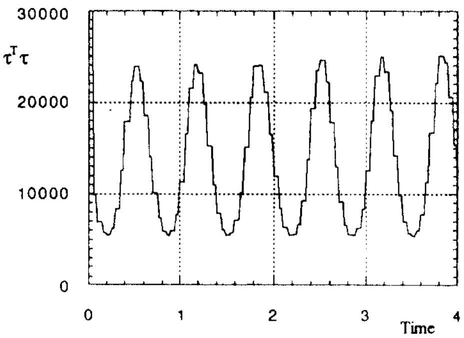

Fig. 8 Torque norm trajectory of null space damped JTM(K_d=200)

in the multiple cooperating redundant manipulator case, compared to the results of the undamped JTM scheme. However, the motion and torque trajectories of the undamped NJM(Figs. (9) ~(11)) reveal, via the increases in the velocity,

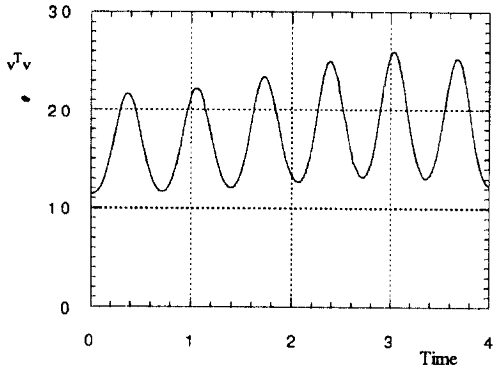

Fig. 9 Velocity norm trajectory of undamped NJM(K_d=0)

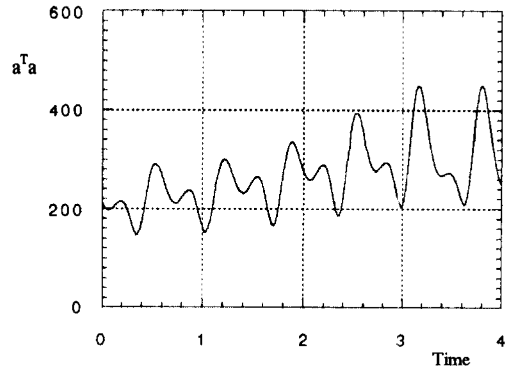

Fig. 10 Acceleration norm trajectory of undamped NJM(K_d=0)

acceleration, and torque norms, that they are deteriorating the desired system performance, even though at a slow rates. Therefore, the stability of algorithms using the transpose of Jacobian among load distribution schemes can not be guaranteed in case of redundant situation, in order to map the optimally distributed operational forces(or contact forces/moments) to each individual manipulator. Such load distribution schemes might include the all of the first category in case of heavy object load and some of the third one in Sec. 1.

The elimination of the stability problem is accomplished by adding fictitious dissipating forces, without affecting the operational forces, to the undamped solutions. Figs. (6)~(8) and Figs. (12)~(14) demonstrate the effectiveness of this approach. From the comparison of those results,

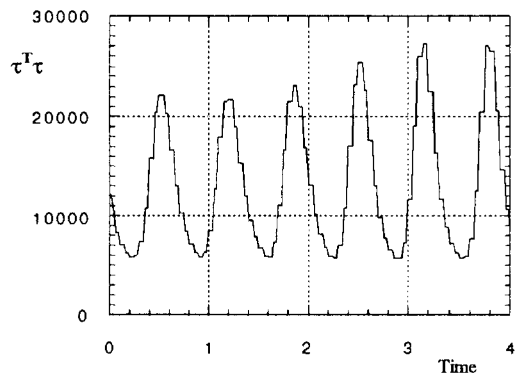

Fig. 11 Torque norm trajectory of undamped NJM($K_d=0$)

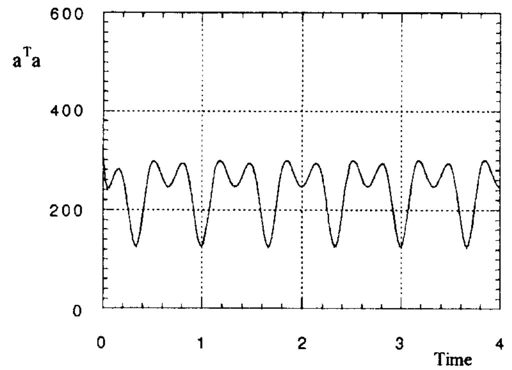

Fig. 13 Acceleration norm trajectory of null space damped NJM($K_d=5$)

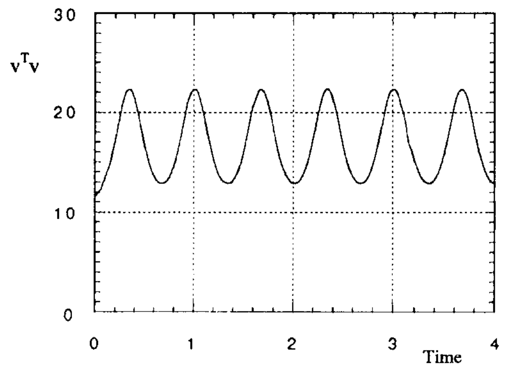

Fig. 12 Velocity norm trajectory of null space damped NJM($K_d=5$)

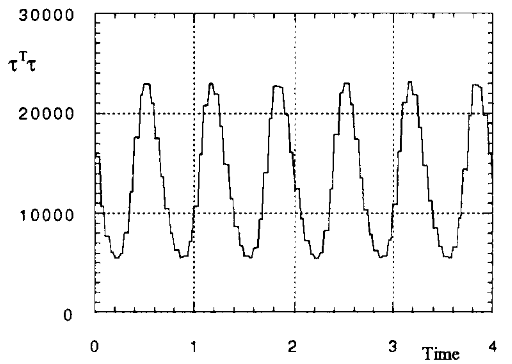

Fig. 14 Torque norm trajectory of null space damped NJM($K_d=5$)

it is concluded that the null space damped natural joint motion(NDNJM) scheme is superior to the null space damped joint torque minimization(NDJTM) scheme with respect to the global torque minimization capability and the resulting stability. In addition, the NDJTM scheme had a much smaller range of damping gains yielding a stable response than did the NDNJM scheme, and also, led to a torque peak and high speed joint motion in its initial stage. Those problems were also observed in the NDJTM scheme for single redundant manipulators. Therefore, the NDNJM scheme is deemed more appropriate for joint torque optimization of the multiple cooperating redundant manipulator system.

6. Conclusion

In this paper, joint torque optimization for multiple cooperating redundant manipulators rigidly handling a common object was considered. This work focused on finding the optimal and stable distribution scheme of the operational forces of a multiple redundant manipulator system to the individual manipulators. Two undamped/damped joint torque optimization schemes(local joint torque minimization and natural joint motion) were formulated and compared with respect to the global torque minimization capability and the resulting stability. From the simulation results of a system of three cooperating redundant manipulators, the null space damped natural joint motion(NDNJM) scheme is deemed best among the considered schemes for the desired system performance. Additionally, the condition for the distribution of the operational forces required to drive the given system along natural joint motion trajectory was addressed. The tendency to instability of natural joint motion was shown that the direct use of the transpose of Jacobian, in order to map the optimally distributed operational forces(or contact forces/moments) to each individual manipulator, seems to be dangerous in cased of redundant situation.

Acknowledgement

This study was supported by the Korea Science and Engineering Foundation(KOSEF) under Grant Number 931-1000-015-2, which is gratefully acknowledged.

APPENDIX

1. Operational Space Dynamic Modeling of Multiple Cooperating Redundant Manipulators Including Joint Motor Stiffness

Assume that the actuated joints in the multiple cooperating redundant manipulator have their own joint motor stiffness, which is solely dependent on the joint motor characteristics. The stiffness matrix of the i^{th} manipulator can be modeled as $[{}_iK^*_{\phi\phi}]$. Including the effects of the stiffness, the equations of motion of the given system, constrained by a predefined operational space trajectory, can be obtained by globally minimizing the constrained Lagrangian. Using the two $m\times 1$ vectors of Lagrange multipliers, λ_i and η_i, respectively, for the i^{th} manipulator, the constrained Lagrangian, L, can be written as

$$L=T-U1-U2+\sum_{i=1}^{L}\lambda_i^T(f_i(\phi_i)-u) + \sum_{i=1}^{L}\eta_i^T(J_i\delta\phi_i-\delta u) \qquad \text{(A1-1)}$$

where the kinetic energy of the given system is given by

$$T=\sum_{i=1}^{L}\frac{1}{2}\dot{\phi}_i^T[{}_iI^*_{\phi\phi}]\dot{\phi}_i \qquad \text{(A1-2)}$$

and the potential energy due to joint deflections, $\delta\phi_i$, is

$$U1=\sum_{i=1}^{L}\frac{1}{2}\delta\phi_i^T[{}_iK^*_{\phi\phi}]\delta\phi_i, \qquad \text{(A1-3)}$$

and

$U2$ = potential energy from gravity forces, respectively.

The objective function, I, of the current optimization problem is given by

$$I=\int_{t_0}^{t}Ldt. \qquad \text{(A1-4)}$$

Using the Calculus of Variations, the necessary conditions for minimizing I are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\phi}_i}\right)-\left(\frac{\partial L}{\partial\phi_i}\right)=0,\ \left(\frac{\partial L}{\partial\lambda_i}\right)=0,$$
$$\left(\frac{\partial L}{\partial\delta\phi_i}\right)=0 \text{ and } \left(\frac{\partial L}{\partial\eta_i}\right)=0,$$
$$i=1,2,\cdots,L \qquad \text{(A1-5)}$$

By substituting the constrained Lagrangian in Eq.(A1-1) into the conditions in (A1-5) and making some simplifications, the necessary conditions in Eq.(A1-5) can be written as

$$[{}_iI^*_{\phi\phi}]\ddot{\phi}_i+C(\phi_i,\dot{\phi}_i)-\left(\left(\frac{\partial J_i}{\partial\phi_i}\right)\delta\phi_i\right)\eta_i=J_i^T\lambda_i,$$
$$i=1,2,\cdots,L \qquad \text{(A1-6)}$$

where

$$C(\phi_i,\dot{\phi}_i)=\left(\frac{d}{dt}[{}_iI^*_{\phi\phi}]\right)\dot{\phi}_i - \dot{\phi}_i^T\left(\frac{1}{2}\frac{\partial[{}_iI^*_{\phi\phi}]}{\partial\phi_i}\right)\dot{\phi}_i+\left(\frac{\partial U2}{\partial\phi_i}\right)$$
$$J_i\ddot{\phi}_i=\ddot{u}-H(\phi_i,\dot{\phi}_i) \quad i=1,2,\cdots,L \qquad \text{(A1-7)}$$

$$[{}_iK^*_{\phi\phi}]\delta\phi_i=J_i^T\eta_i \quad i=1,2,\cdots,L \qquad \text{(A1-8)}$$

and

$$J_i\delta\phi_i=\delta u \quad i=1,2,\cdots,L \qquad \text{(A1-9)}$$

Adding Eq.(A1-8) to (A1-6) yields

$$[{}_iI^*_{\phi\phi}]\ddot{\phi}_i+C(\phi_i,\dot{\phi}_i) + [{}_iK^*_{\phi\phi}]\delta\phi_i-\left(\left(\frac{\partial J_i}{\partial\phi_i}\right)\delta\phi_i\right)\eta_i=J_i^T(\lambda_i+\eta_i),$$
$$i=1,2,\cdots,L \qquad \text{(A1-10)}$$

Multiplying both sides by $J_i[{}_iI^*_{\phi\phi}]^{-1}$ yields

$$J_i\ddot{\phi}_i+J_i[{}_iI^*_{\phi\phi}]^{-1}\{C(\phi_i,\ \dot{\phi}_i)+[{}_iK^*_{\phi\phi}]\delta\phi_i-\left(\left(\frac{\partial I_i}{\partial\phi_i}\right)\delta\phi_i\right)\eta_i\}=J_i[{}_iI^*_{\phi\phi}]^{-1}J_i^T(\lambda_i+\eta_i). \quad \text{(A1-11)}$$

Replacing $J_i\ddot{\phi}_i$ with Eq.(A1-7) yields

$$\ddot{u}-H(\phi_i,\ \dot{\phi}_i)+J_i[{}_iI^*_{\phi\phi}]^{-1}\{C(\phi_i,\ \dot{\phi}_i)+[{}_iK^*_{\phi\phi}]\delta\phi_i-\left(\left(\frac{\partial I_i}{\partial\phi_i}\right)\delta\phi_i\right)\eta_i\}=J_i[{}_iI^*_{\phi\phi}]^{-1}J_i^T(\lambda_i+\eta_i). \quad \text{(A1-12)}$$

Multiplying both sides by $(J_i[{}_iI^*_{\phi\phi}]^{-1}J_i^T)^{-1}$ and using the transpose of the pseudo-inverse,

$({}_iJ_i^+)^T=(J_i[{}_iI^*_{\phi\phi}]^{-1}J_i^T)^{-1}J_i[{}_iI^*_{\phi\phi}]^{-1}$, yields

$$(\lambda_i+\eta_i)=(J_1[{}_iI^*_{\phi\phi}]^{-1}J_i^T)^{-1}(\ddot{u}-H(\phi_i,\ \dot{\phi}_i))+({}_iJ_i^+)^TC(\phi_i,\ \dot{\phi}_i)+({}_iJ_i^+)^T[{}_iK^*_{\phi\phi}]\delta\phi_i-({}_iJ_i^+)^T\left(\left(\frac{\partial J_i}{\partial\phi_i}\right)\delta\phi_i\right)\eta_i,\quad i=1,\ 2,\ \cdots,\ L \quad \text{(A1-13)}$$

Using Eq.(A1-8), with the assumption that $[{}_iK^*_{\phi\phi}]$ be a nonsingular matrix, the third term of Eq. (A1-13) becomes

$$({}_iJ_i^+)^T[{}_iK^*_{\phi\phi}]\delta\phi_i=({}_iJ_i^+)^T[{}_iK^*_{\phi\phi}][{}_iK^*_{\phi\phi}]^{-1}J_i^T\eta_i=\eta_i. \quad \text{(A1-14)}$$

Thus, the operational forces of the i^{th} manipulator, F_i, can be interpreted as

$$F_i=\lambda_i+\eta_i=[{}_iI^*_{uu}]\ddot{u}+{}_iC_u+{}_iS_u \quad \text{(A1-15)}$$

where the operational inertia matrix of the i^{th} manipulator is

$$[{}_iI^*_{uu}]=(J_i[{}_iI^*_{\phi\phi}]^{-1}J_i^T)^{-1} \quad \text{(A1-16)}$$

and the operational Coriolis, centripetal and gravity force vector of the i^{th} manipulator is

$${}_iC_u=({}_iJ_i^+)^TC(\phi_i,\ \dot{\phi}_i)-[{}_iI^*_{uu}]H(\phi_i,\ \dot{\phi}_i) \quad \text{(A1-17)}$$

with the operational spring forces of the ith manipulator being

$${}_iS_u=({}_iJ_i^+)^T([{}_iK^*_{\phi\phi}]\delta\phi_i-\left(\left(\frac{\partial I_i}{\partial\phi_i}\right)\delta\phi_i\right)({}_iJ_i^+)^T[{}_iK^*_{\phi\phi}]\delta\phi_i)=({}_iJ_i^+)^T{}_iS_\phi. \quad \text{(A1-18)}$$

The operational forces of the total multiple cooperating manipulator system can therefore be expressed as

$$F=\sum_{i=1}^{L}F_i=[I^*_{uu}]\ddot{u}+C_u+S_u \quad \text{(A1-19)}$$

where the effective inertia of the given system is

$$[I^*_{uu}]=\sum_{i=1}^{L}(J_i[{}_iI^*_{\phi\phi}]^{-1}J_i^T)^{-1} \quad \text{(A1-20)}$$

and the operational Coriolis, centripetal and gravity forces of the given system is

$$C_u=\sum_{i=1}^{L}\{({}_iJ_i^+)^TC(\phi_i,\ \dot{\phi}_i)-[{}_iI^*_{uu}]H(\phi_i,\ \dot{\phi}_i)\} \quad \text{(A1-21)}$$

with the operational spring forces of the given system being

$$S_u=\sum_{i=1}^{L}({}_iJ_i^+)^T{}_iS_\phi \quad \text{(A1-22)}$$

Rewriting Eqs. (A1-13) and (A1-15) with $J_i\ddot{\phi}_i=J_i[{}_iI^*_{\phi\phi}]^{-1}({}_i\tau_\phi-C(\phi_i,\ \dot{\phi}_i)-{}_iS_\phi)$, the unique functional relationship which maps the joint torque set of the i^{th} manipulator into the corresponding operational forces is expressed as

$$F_i=({}_iJ_i^+)^T{}_i\tau_\phi. \quad \text{(A1-23)}$$

According to Eq. (A1-19), the unique functional relationship which maps the joint torque set of the total system into the operational forces is expressed as

$$F=\sum_{i=1}^{L}F_i=\sum_{i=1}^{L}({}_iJ_i^+)^T{}_i\tau_\phi=[J_i^+]^T\tau_\phi \quad \text{(A1-24)}$$

with $[J_i^+]^T=[({}_1J_i^+)^T:\cdots:({}_iJ_i^+)^T:\cdots({}_LJ_i^+)^T]$ and $\tau_\phi=[{}_1\tau_\phi^T\cdots{}_i\tau_\phi^T\cdots{}_L\tau_\phi^T]^T$.

2. Effectively Equivalent Operational Stiffness Matrix of Multiple Cooperating Redundant Manipulators

From Eqs.(A1-14) and (A1-18), the effective operational spring force of the i^{th} manipulator, F_{is}, is given by

$$F_{is}=\eta_i-({}_iJ_i^+)^T\left(\left(\frac{\partial J_i}{\partial\phi_i}\right)\delta\phi_i\right)\eta_i. \quad \text{(A2-1)}$$

Solving for η_i from Eqs.(A1-8) and (A1-9) yields

$$\eta_i=(J_i[{}_iK^*_{\phi\phi}]^{-1}J_i^T)^{-1}\delta u=[{}_iK^*_{uu}]_L\delta_u \quad \text{(A2-2)}$$

where η_i and $[{}_iK^*_{uu}]_L$ are the effective operational linear spring force and the effective operational linear stiffness matrix, respectively, due to the joint motor stiffness, $[{}_iK^*_{\phi\phi}]$, of the i^{th} manipulator.

Employing the second order KIC matrix and the generalized scalar product(°) seen in Freeman(1988), the second term of Eq. (A2-1) can be written as

$$(J_i^+)^T\left(\left(\frac{\partial J_i}{\partial \phi_i}\right)\delta\phi_i\right)\eta_i = (J_i^+)^T(\eta_i^T\circ[{}_iH^u_{\phi\phi}])\delta\phi_i. \quad \text{(A2-3)}$$

The right hand side of Eq. (A2-3) might be interpreted as nonlinear spring force mapped in operational space of joint nonlinear spring forces, $(\eta_i^T\circ[{}_iH^u_{\phi\phi}])\delta\phi_i$ like mapping the centrifugal, Coriolis and gravity forces to the corresponding operational forces. The nonlinear joint spring force might result from the antagonistic effect of the manipulator nonlinear geometry between the effective joint spring torques and the physical spring forces. Therefore, the antagonistic joint stiffness is defined as

$$[{}_iK^*_{\phi\phi}]_A = (\eta_i^T\circ[{}_iH^u_{\phi\phi}]). \quad \text{(A2-4)}$$

Combining Eqs.(A1-8) and (A2-2), the infinitesimal displacement relationship between $\delta\phi_i$ and du can be expressed as

$$\delta\phi_i = [{}_iK^*_{\phi\phi}]^{-1}J_i^T[{}_iK^*_{uu}]_L\delta u. \quad \text{(A2-5)}$$

Then, the effective operational spring force in Eq. (A2-1) becomes

$$F_{is} = [{}_iK^*_{uu}]\delta u = ([{}_iK^*_{uu}]_L + (J_i^+)(\eta_i^T\circ[{}_iH^u_{\phi\phi}])[{}_iK^*_{\phi\phi}]^{-1}J_i^T[{}_iK^*_{uu}]_L)\delta u. \quad \text{(A2-6)}$$

The operational spring force of the total system can be obtained by adding the contributions of the individual manipulators as

$$F_s = [K^*_{uu}]\delta u = \sum_{i=1}^{L} F_{is} = (\sum_{i=1}^{L}[{}_iK^*_{uu}])\delta u. \quad \text{(A2-7)}$$

Therefore, the effectively equivalent operational stiffness matrix of the multiple cooperating redundant manipulator system can be written as

$$[K^*_{uu}] = \sum_{i=1}^{L}[{}_iK^*_{uu}] = \sum_{i=1}^{L}(J_i[{}_iK^*_{\phi\phi}]^{-1}J_i^T) + \sum_{i=1}^{L}(J_i^+)^T(\eta_i^T\circ[{}_iH^u_{\phi\phi}])[{}_iK^*_{\phi\phi}]^{-1} J_i^T(J_i[{}_iK^*_{\phi\phi}]^{-1}J_i^T)^{-1}. \quad \text{(A2-8)}$$

If δu is small enough, the nonlinear stiffness force in Eq.(A2-1) can be neglected, since the forces are proportional to$(\delta u)^2$. And then, the effective operational stiffness in Eq.(A2-6) can be replaced by the effective linear operational stiffness:

$$[K^*_{uu}] = \sum_{i=1}^{L}[{}_iK^*_{\phi\phi}]_L = \sum_{i=1}^{L}(J_i[{}_iK^*_{\phi\phi}]^{-1}J_i^T)^{-1}. \quad \text{(A2-9)}$$

The derivation above for the effectively equivalent operational stiffness allows the physical interpretation of the operational force distribution scheme shown in Eq. (38) of this paper.

References

Carignan, C. R. and Akin, D. L., June, 1989, "Optimal Force Distribution for Payload Positioning Using a Planar Dual-Arm Robot," *J. of Dynamic Syst., Measurement and Control*, Vol. 111, pp. 205~210.

Freeman, R. A., 1988, "Freedforward Stiffness Control of Overconstrained Mechanisms/Robotic Linkage System," *ASME winter annual meeting.*

Hsu, P., 1989, "Control of Multi-manipulator Systems — Trajectory Tracking, Load Distribution, Internal Force Control and Decentralized Architecture," *IEEE Int. Conf. on Robotics and Automation*, Vol. 2, pp. 1234~1239.

Hu, Y. R. and Goldenberg, A. A., 1990, "Dynamic Control of Multiple Coordinated Redundant Manipulators with Torque Optimization," *IEEE Int. Conf. on Robotics and Automation*, Vol. 2, pp. 1000~1005.

Kang, H. J. and Freeman, R. A., 1993, "Null Space Damping Method for Local Joint Torque Optimization of Redundant Manipulators," *Journal of Robotic Systems*, Vol. 10, No. 2, pp. 249~270.

Kang, H. J. and Freeman, R. A., 1994, "Evaluation of Loop Constraints for Kinematic and Dynamic Modeling of General Closed-Chain Robotic Systems," *KSME Journal*, Vol. 8, No. 2, pp. 115~126.

Kerr, J. and Roth, B., 1986, "Analysis of Multifingered Hands," *International J. of*

Robotics Research, Vol. 4-4, pp. 3~17.

Khatib, O., 1988, "Augmented Object and Reduced Effective Inertia in Robot Systems," *Proc. of American Control Conference*, pp. 2140~2147.

Kreutz, K. and Lokshin, A., 1988, "Load Balancing and Closed Chain Multiple Arm Control," *ASME Control Conf.*, pp. 2148~2155.

Kumars, V. and Waldron, K., 1987, "Suboptimal Algorithms for Force Distribution in Multi-Fingered Robot Hands," *IEEE Int. Conf. on Robotics and Automation*, pp. 252~257.

Nedungadi, A. and Kazerounian, K., 1989, "A Local Solution with Global Characteristics for the Joint Torque Optimization of a Redundant Manipulator," *Journal of Robotic Systems*, Vol. 6, No. 5, pp. 631~654.

Pittelkau, M. E., 1988, "Adaptive Load-Sharing Force Control for Two-Arm Manipulators," *IEEE Int. Conf. on Robotics and Automation*, pp. 498~503.

Zheng, Y. F. and Luh, J. Y. S., 1988, "Optimal Load Distribution for Two Industrial Robots Handling a Single Object," *IEEE Int. Conf. on Robotics and Automation*, Vol. 1, pp. 344~349.